MAT 1341 C, Quiz 3

March 11, 2019

Length: 15 minutes.

Professor: Rachid Bentoumi.

Solution

Family name:______________________

First name:______________________

Student number:____________________

1	D
2	A
3	E
Total	

PLEASE CAREFULLY READ THESE INSTRUCTIONS:

1. Carefully read each question and **record your responses in the space provided on this page as well as the question page.**
2. You are not allowed to consult your notes or any books. Calculators, phones, and other electronic devices are not allowed.
3. There are three multiple choice questions, each worth 1 point. No partial credit will be awarded. **You must indicate the method you used to select the correct answer; unjustified answers will not be given credit.**

   Record your answers both on the question page and on the title page.

1. If A is an $n \times 2$ matrix and $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then the second column of the matrix AB is

A. only defined if $n = 2$.

B. the same as the second column of A.

C. the same as the second column of B.

D. the same as the first column of A.

E. the same as the first column of B.

F. the sum of the first and second columns of A.

Solution: A is an $n \times 2$ matrix $\Rightarrow$

$$A = [c_1 \; c_2], \text{ where } c_1, c_2 \in \mathbb{R}^n$$

Now,

$$AB = [c_1 \; c_2] \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = [(1)c_1 + (1)c_2 \quad (1)c_1 + (0)c_2]$$

$$= [(c_1 + c_2) \quad c_1]$$

The second column of the matrix AB is the same as the first column of A

Answer: D

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$, what is the second row of A^{-1}?

A. $(-\frac{15}{8}, \frac{1}{2}, \frac{3}{8})$

B. $(-\frac{15}{4}, 1, \frac{3}{4})$

C. $(\frac{13}{8}, -\frac{1}{2}, -\frac{1}{8})$

D. $(-\frac{1}{2}, \frac{1}{2}, 0)$

E. $(-\frac{15}{8}, \frac{1}{8}, \frac{3}{8})$

F. $(-15, 4, 3)$

Solution: A is an 3 x 3 matrix

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 4 & \frac{5}{4} & 0 & -1/4 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 5/4 & 0 & -\frac{1}{4} \end{array}\right]$$

$$\sim \underbrace{\left[\begin{array}{ccc} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right.}_{I_3} \underbrace{\left|\begin{array}{ccc} 1 & -1/2 & 0 \\ \boxed{-15/8 \quad 1/2 \quad 3/8} & & \\ 5/4 & 0 & -1/4 \end{array}\right]}_{A^{-1}} \leftarrow \text{second row of } A^{-1}$$

Answer: A

3. Consider the following matrix

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -4 & 2 \end{bmatrix}.$$

Which of the following is a basis for $\ker(A)$?

A. $\{(1,0,0),\ (0,1,0)\}$

B. $\{(1,2,0)\}$

C. $\{(-1,1,0)\}$

D. $\{(-1,2,2),\ (2,-4,2)\}$

E. $\{(2,1,0)\}$

F. $\{(1,0,0),\ (0,1,0),\ (0,0,1)\}$

Solution: We need to solve $Ax=0$ $([A|0])$

$$[A|0] = \left[\begin{array}{ccc|c} -1 & 2 & 2 & 0 \\ 2 & -4 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 1 & -2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

* x_1 and x_3 are leading variables
* $(x_2 = t)$ is non-leading variables (parameters)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2s \\ s \\ s \end{bmatrix} = s\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\operatorname{Ker}(A) = \left\{ s\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \,\middle|\, s \in \mathbb{R} \right\} = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\} = \operatorname{span}\{(2,1,0)\}$$

Answer: E

$\{(2,1,0)\}$ is a basis for Ker(A) since it is L.I and spans Ker (A)